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Dynamics of the dislocation damping

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A quantitative theory of dynamic loss of damping due to dislocation is developed by the powerful operational method of Heaviside (1966). The problem is based on the vibrating string model of dislocation. The attenuation resulting from dislocation motion and viscous damping is calculated in two different cases :
a) When frequency is large.
b) When frequency is low.
The theoretical graphs for normalized decrement *vs* normalized frequency are drawn for both high and low frequencies. The graphs show a good agreement with the works of previous experimental and theoretical workers.

1. INTRODUCTION

The attenuation of sound waves in crystals depends strongly on the motion of dislocations. The dynamics of dislocation motion depends upon the interaction of dislocation with point defects, phonons, electrons and other dislocations.

The quantitative theory of the dislocation internal friction effect is rationalized in the terms of the vibrating string model of dislocation suggested by a large number of scientists.

Mott (1952) assumed that a pure single crystal contains before deformation a network of dislocations and dislocation segments between network points bow out in phase with an applied external oscillatory stress. Friedel (1956) extended this model by assuming that impurity atoms as well as dislocation network intersections can pin down dislocations. Eshelby (1956) introduced the idea of the effect of inertia and viscosity on dislocation motion. Koehler (1952) considered a dislocation line pinned down with impurities and distributed at random. He suggested that the observed amplitude dependence of the damping results from the pulling away of the dislocations from the impurity pinning points.

General quantitative theory of dislocation motion on the basis of the vibrating string model was first given by Granato & Lücke (1956). They considered a dislocation line with two types of pinning points, namely nodes of the dislocation network on the one hand, and impurities and other point imperfections on the other hand. At low strain amplitudes one has only a vibration of the dislocation segment in between the pinning points and therefore a loss only due to viscosity of dislocation motion.

We recall the following assumptions in developing the dislocation theory :

1) The interaction of the dislocation with the lattice has been neglected.

2) The interaction between the dislocations has also been negected.

3) Length of the dislocation loops is assumed invariant.

4) We consider a small fraction of dislocation which lies nearly parallel to the Peierls valleys because we know that dislocations have their least energy when they lie in Peierls valleys.

5) Peierls stress is neglected in the string model.

2. Explanation of the Symbols

A = Effective mass per unit length $= \rho a^2$
B = Damping constant.
C = Effective tension on the dislocation line $= \dfrac{2Ga^2}{\pi(1-\nu')}$ where ν' = Poisson's ratio, a = Burger's vector.
σ = The applied stress.
ρ = Density of the material.
ϵ = Total strain produced by applied stress.
t = Variable time.
G = Shear modulus for the material.
Λ = Dislocation density.
l = Length of a dislocation loop
ξ = Displacement of the dislocation from equilibrium position
y = Co-ordinate of an element of the dislocation line
x = Variable co-ordinate along which the stress is applied.
α = Attenuation constant.
ω = Frequency of the oscillating applied stress.
v = Velocity of the elastic wave.
$\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}t}$ (Differential operator).

3. Solution of the Problem

The decrement and apparent modulus change felt by a stress travelling through a solid which contains pinned dislocations may be found by applying Newton's Law to the system to obtain the equation of motion which is given by

$$\frac{\partial^2\sigma}{\partial x^2} - \rho\frac{\partial^2\epsilon}{\partial t^2} = 0 \qquad \text{... (1.0)}$$

The strain ϵ is made up of two kinds of strain (1) dislocation strain ϵ_{dis} due to motion of dislocation, (2) the elastic strain ϵ_{el} under the influence of the applied stress σ, *i.e.*,

$$\epsilon = \epsilon_{el}+\epsilon_{dis} \qquad (2.0)$$

The elastic strain is given by

$$\epsilon_{el} = \frac{\sigma}{G} \qquad (3.0)$$

This dislocation strain produced by a loop of length l is given by

$$\epsilon_{dis} = \Lambda a\bar{\xi} = \frac{\Lambda a}{l} \int_0^l \xi \mathrm{d}y \qquad (4.0)$$

From equations (1.0) to (4.0) we get

$$\frac{\partial^2\sigma}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2\sigma}{\partial t^2} = \frac{\Lambda\rho a}{l}\frac{\partial^2}{\partial t^2}\int_0^l \xi \mathrm{d}y \qquad (5.0)$$

The displacement ξ of the dislocation, under the influence of an applied stress ξ given by the mathematical model for the equation of motion of a pinned-down dislocation loop used by Koehler (1952) in the light of Rayleigh's stretched string theory, may be expressed as :

$$A\frac{\partial^2\xi}{\partial t^2}+B\frac{\partial\xi}{\partial t}-C\frac{\partial^2\xi}{\partial y^2} = a\sigma \qquad \ldots (6.0)$$

where ξ is a function of x, y and t, and the term on the right is the force per unit length exerted on the dislocation by the external shearing stress.

The boundary conditions for all values of x and t are given by

$$\left.\begin{array}{ll} \text{at } y = 0, & \xi = 0 \\ \text{and at } y = l & \xi = 0 \end{array}\right\} \qquad \ldots (6.1)$$

The equation (6.0) in operational form is

$$\frac{\partial^2\xi}{\partial y^2} - \frac{\mathrm{D}^2+d\mathrm{D}}{C_1^2}\xi = -\frac{a}{c}\sigma \qquad \ldots (6.2)$$

where $d = \frac{B}{A}$ and $C_1^2 = C/A$.

Let us assume that applied stress σ is periodic in time and independent of y and let

$$\sigma = \sigma_0 \exp(-\alpha x)\exp[i\omega(t-x/v)] \qquad \ldots (7.0)$$

By substituting (7.0) in (6.2) we have from equation (6.2)

$$\frac{\partial^2\xi}{\partial y^2} - \frac{\mathrm{D}_1^2}{C_1^2}\xi = -k\exp(i\omega t) \qquad \ldots (6.3)$$

where $D_1^2 = \mathrm{D}^2+d\mathrm{D}$ and $k = \frac{a}{c}\sigma_0\exp\left(-\alpha x-\frac{i\omega x}{v}\right)$.

The solution of equation (6.3) is given by

$$\xi = A_1 \cosh \frac{D_1}{C_1} y + B \sinh \frac{D_1}{C_1} y + \frac{kC_1^2}{D_1^2} \exp(i\omega t). \qquad \ldots \quad (6.4)$$

where A_1 and B_1 are two unknown constants to be determined from boundary conditions.

Using equations (6.1) and (6.4), the equation (6.4) can be rewritten as

$$\xi = \frac{kC_1^2}{D_1^2}\left(1-\cosh \frac{D_1}{C_1} y + \tanh \frac{D_1}{2C_1} l \,.\, \sinh \frac{D_1}{C_1} y \right) \exp(i\omega t),$$

$$\text{or, } \xi = \frac{kC_1^2}{D_1^2}\Big[1-\exp\left(-\frac{D_1}{C_1} y\right)+\exp\left\{-\frac{D_1}{C_1}(y+l)\right\}-\exp\left\{-\frac{D_1}{C_1}(y+2l)\right\}$$

$$+\ldots-\exp\left\{-\frac{D_1}{C_1}(l-y)\right\}+\exp\left\{-\frac{D_1}{C_1}(2l-y)\right\}-\ldots\Big] \exp(i\omega t) \quad \ldots \quad (6.5)$$

The operational solution of equation (6.5) yields

$$\xi = \mu[1-\exp(-\nu y)+\exp\{-\nu(y+l)\}-\exp\{-\nu(y+2l)\}+\ldots-\exp\{-\nu(l-y)\}$$

$$+\exp\{-\nu(2l-y)\}-\exp\{-\nu(3l-y)\}+\ldots] \quad \exp(i\omega t)$$

or
$$\xi = \mu\left[1-\frac{\cosh \nu y+\cosh \nu(y-l)}{1+\cosh \nu l}\right] \exp(i\omega t) \qquad \ldots \quad (6.6)$$

where $\quad \nu = \dfrac{(i\omega+\frac{1}{2}d)}{C_1}$ and $\mu = -kC_1^2\left[\dfrac{1}{d^2+\omega^2}+\dfrac{id}{\omega(d^2+\omega^2)}\right]$.

This is the most general solution of the displacement of the dislocation loop of length l.

Case—1. At high frequency $\nu \gg 1$ will be large so that $\exp\left(-\frac{\nu l}{2}\right)$ and $\exp\left(-\frac{\nu y}{2}\right)$ are very small, and the equation (6.6) becomes

$$\xi = \mu \exp(i\omega t). \qquad \ldots \quad (6.7)$$

The equation (6.7) shows that ξ is independent of y. This shows that at such high frequencies the dislocations do not change in configuration but oscillate like rigid bars. Now from equations (5.0) and (6.7) we have

$$\left(\alpha+\frac{i\omega}{v}\right)^2+\frac{\rho\omega^2}{G}=\frac{i\Lambda\rho a^2\omega d}{A(d^2+w^2)} \qquad \ldots \quad (8.0)$$

Equating imaginary terms from both sides of equation (8.0) and simplifying

$$\alpha = \frac{1}{2v}\left(\frac{\pi}{8}\right)\eta^2\Delta_0\Lambda\,\frac{d}{d^2+\omega^2} \qquad \dots \quad (9.0)$$

where $\eta^2 = \pi^2 C/A$ and $\Delta_0 = \dfrac{8Ga^2}{\pi^3 C}$.

The decrement, *i.e.* attenuation per wave length is

$$\Delta = \lambda\alpha = \alpha\,.\,\frac{2\pi v}{\omega} = \left(\frac{\pi^2}{8}\right)\eta^2\Delta_0\Lambda\,\frac{d}{\omega(d^2+\omega^2)}\,. \qquad \dots \quad (10.0)$$

Equation (10.0) when written in terms of normalized decrement and normalized frequency is given by

$$\frac{\Delta}{\left(\frac{\pi^2}{8}\right)\Delta_0\Lambda l^2} = \frac{\left(\frac{d}{\omega_0}\right)}{\left(\frac{\omega}{\omega_0}\right)\left[\left(\frac{d}{\omega_0}\right)^2+\left(\frac{\omega}{\omega_0}\right)^2\right]} \qquad \dots \quad (10.1)$$

The graphical representation of equation (10.1) is given in figure 1.

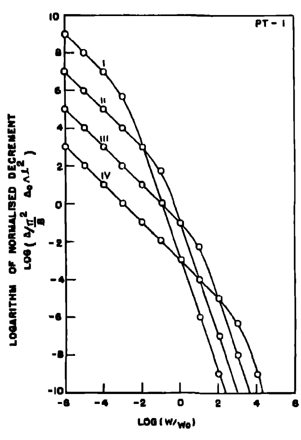

Figure 1. The logarithm of normalized decrement is plotted against the logarithm of normalized frequency, at high frequency range from equation (10.1) for different values of (d/ω_0).

I — $\frac{d}{\omega_0} = 10^{-3}$, II — $\frac{d}{\omega_0} = 10^{-1}$ III — $\frac{u}{\omega_0} = 10^{1}$ IV — $\frac{a}{\omega_0} = 10^{3}$.

The factor ($\pi^2/8$) automatically appears in equation (10.0) in dealing with high frequency cases where the dislocations vibrate as rigid rods as has been remarked under equation (6.7). It is not surprising that in dealing with high freequency cases one has to take large number of terms in accordance to Granato & Lücke's (1956) treatment where they have remarked the appearance of the same factor ($\pi^2/8$) for summing an infinite series. In this context the present method is more elegant and straightforward, than the Granato & Lücke-method, and gives the correct result without any speculation.

Case—2 :

When frequency is low, *i.e.*, when $\nu \ll 1$, the equation (6.6) is

$$\xi = \mu\left[1-\frac{\cosh \nu y+\cosh \nu(y-l)}{1+\cosh \nu l}\right]\exp(i\omega t) \quad \ldots \quad (6.6)$$

or

$$\xi = \mu\left[1-\frac{\cosh \nu(y-l/2)}{\cosh(\nu l/2)}\right]\exp(i\omega t) \quad \ldots \quad (6.8)$$

Now $\int_0^l \xi dy$ which appears in the right hand side of equation (5.0) when evaluated for low frequencies using equation (6.8) for ξ gives

$$\int_0^l \xi\, dy = \frac{\mu\nu^2 l^3}{12}\exp(i\omega t) \quad \ldots \quad (6.9)$$

From equations (5.0) and (6.9) we have

$$\left(\alpha+\frac{i\omega}{v}\right)^2\sigma+\frac{\rho}{G}\,\omega^2\sigma = \frac{\Lambda\rho a}{l}(-\omega^2)\,\frac{\mu l^3}{12}\left[\left(\frac{d}{2C_1}\right)^2-\frac{\omega^2}{C_1^{\,2}}+\frac{id\omega}{C_1^{\,2}}\right]\times\exp(i\omega t)$$

Equating imaginary terms from both sides we have

$$\alpha = \left(\frac{v}{2}\right)\frac{\Lambda\rho a^2 l^2}{12C}\,\frac{\omega^2 d}{d^2+\omega^2} \quad \ldots \quad 11.0)$$

Thus the decrement is given by

$$\Delta = \left(\frac{\pi^4}{96}\right)\Delta_0\Lambda l^2\,\frac{\omega d}{\omega^2+d^2} \quad \ldots \quad (12.0)$$

Assuming reasonable values of the parameters Δ_0, Λ, l, B and A, the variation of logarithm of frequency *vs* logarithm of decrement is plotted from equation (12.0) in figure 2. From figure 2, the following features are quite apparent ; (1) Decreasing loop length (allowing more irradiation) decreases the decrement ; (2)

Increasing the value of d, *i.e.*, damping, the frequency for the maximum decrement shifts to higher frequency region ; (3) In the low frequency region the dislocations

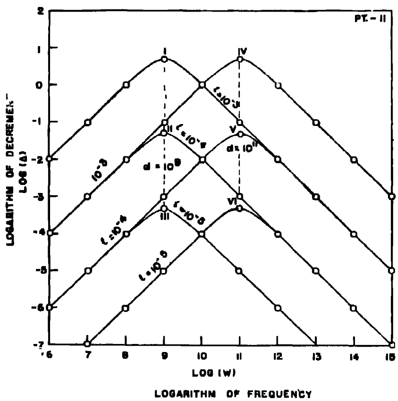

Figure 2. The logarithm of frequency *vs* logarithm of decrement is plotted from equation (12.0).

I—$d = 10^9$, $l = 10^{-3}$, II—$d = 10^9$, $l = 10^{-4}$, III—$d = 10^9$, $l = 10^{-5}$,

IV—$d = 10^{11}$ $l = 10^{-3}$, V—$d = 10^{11}$, $l = 10^{-4}$ VI—$d = 10^{11}$, $l = 10^{-5}$

of the lower loop lengths in the specimen having lower values of d, behave in the same way as dislocations having higher loop lengths in specimen of higher d, but in the high frequency region dislocations of lower loop lengths in the specimen having higher values of d, behave in the same way as dislocations having higher loop lengths but lower value of d.

In terms of normalized decrement and normalized frequency the equation (12.0) can be written as

$$\frac{\Delta}{\left(\frac{\pi^4}{96}\right)\Delta_0 \Lambda l^2} = \frac{\left(\frac{d}{\omega_0}\right)\left(\frac{\omega}{\omega_0}\right)}{\left(\frac{\omega}{\omega_0}\right)^2 + \left(\frac{d}{\omega_0}\right)^2} \qquad \ldots \quad (12.1)$$

The graphical representation of equation (12.1) is shown in figure 3. Figure 3 agrees with the theoretical work of Granato & Lcüke (1956).

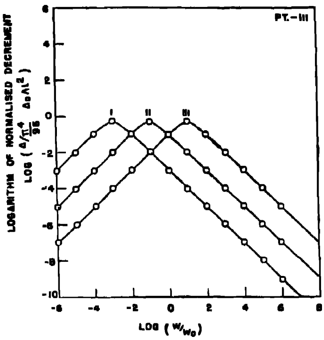

Figure 3. The logarithm of normalized decrement is plotted against the logarithm of normalized frequency at low frequency range from equation (12.1) for different values of (d/ω_0).

$$\text{I}-\frac{d}{\omega_0}=10^{-3},\qquad \text{II}-\frac{d}{\omega_0}=10^{-1},\qquad \text{III}-\frac{d}{\omega_0}=10^{1}.$$

4. Discussion

Dependence of decrement Δ with damping as depicted by equations (10.1) and (12.1) for high and low frequencies can be classified for large and small damping ranges in the following scheme :

(I) $d/\omega_0 \gg 1$ for large damping.

(II) $d/\omega_0 \ll 1$ for small damping.

At high frequencies Δ in equation (10.1) can be written as,

$$\Delta=\left(\frac{\pi^2}{8}\right)\Delta_0\Lambda\,\frac{l^2}{d^2}\left[\frac{\left(\frac{d}{\omega_0}\right)}{\left(\frac{\omega}{\omega_0}\right)}-\frac{\left(\frac{\omega}{\omega_0}\right)\left(\frac{d}{\omega_0}\right)}{\left(\frac{d}{\omega_0}\right)^2+\left(\frac{\omega}{\omega_0}\right)^2}\right] \quad \ldots \quad (10.2)$$

For small damping neglecting the term $(d/\omega_0)^2$,

$$\Delta=0 \quad \ldots \quad (10.3)$$

That is, in high enough frequency and low enough damping where dislocation motion is like that of an unpinned or straight dislocation, the damping vanishes as is evident from equation (10.3). This phenomenon happens in the case of pure material or an annealed metal, *i.e.*, when the material is only elastic.

Again at high frequencies and with high damping equation (10.2) reduces to

$$\Delta = \left(\frac{\pi^2}{8}\right) \Delta_0 \Lambda \frac{l^2}{d} \cdot \frac{1}{\omega} \qquad \ldots \quad (10.4)$$

Equation (10.4) shows that the decrement varies inversely with the first power of frequency. This is again in agreement with the work of Granato & Lücke

After simplifying equation (6.8) we get

$$\xi = \mu \cdot \frac{2 \sinh (\nu\gamma/2) \cdot \sinh \nu/2(l-y)}{\cosh (\nu l/2)} \exp(i\omega t)$$

For small ν, *i.e*, when both frequency and damping is small,

$$\xi = A' y(l-y) \qquad \ldots \quad (13.0)$$

where

$$A' = \frac{\mu\nu^2}{2} \exp(i\omega t).$$

A similar loop shape to that given by equation (13.0) was obtained by Mason (1966) in dealing with attenuation by electron damping.

At low frequencies and at low damping equation (12.1) gives

$$\Delta = \left[\frac{\pi^4}{96}\right] \Delta_0 \Lambda l^2 \frac{d}{\omega} \qquad (12\ 2)$$

i e, the decrement varies inversely with frequency.

And now in low frequency high damping case,

$$\Delta = \left(\frac{\pi^4}{96}\right) \Delta_0 \Lambda l^2 \cdot \frac{\left(\frac{d}{\omega_0}\right)\left(\frac{\omega}{\omega_0}\right)}{\left(\frac{d}{\omega_0}\right)^2 \left[1 + \frac{(\omega/\omega_0)^2}{(d/\omega_0)^2}\right]}$$

$$= \left(\frac{\pi^4}{96}\right) \Delta_0 \Lambda l^2 \left[\frac{\omega}{d}\right] \qquad (12.3)$$

i.e., the decrement varies linearly with frequency. This is again in agreement with Granato & Lücke's theoretical prediction. Reliable measurements by using the widely used pulse technique provide no evidence for the linear frequency dependence of decrement as obtained above. This, however, does not rule out the validity of the KGL model on which the present work is built up if we remember that the specimen may be expected to vary greatly as a result of the pronounced structure-sensibivity of the effects. Also a stress-pulse may be looked upon as a quasi monochromatic wave packet interacting differently with the material as regards frequency and amplitude dependence. This makes decrement measurement etc., increasingly difficult and uncertain and cannot be handled within the frame work of linear response of the system.

REFERENCES

Eshelby, J. D. *Dislocation and Mechanical Properties of Crystal*, Wiley, New York, pp. 91.

Friedel J. 1956 *Dislocation and Mechanical Properties of Crystals*, (J. C. Fisher etc, eds) Wiley, New York, pp. 330-32.

Granato, A. V. & Lucke, K. 1956 *J. Appl Phys* **27**, No. 6 583-93.

Heaviside O. 1966 *Methods of Mathematical Physics* (Cambridge, 3rd Ed.) By Jeffreys, H & & Jefrreys, B. pp. 228-43.

Koehler, J. S. 1952 *Imperfection in Nearly Perfect Crystals* (W. Shockley *et al*, eds.) Wiley, New York, pp 197-221.

Mason, W P. 1966, *Physical Acoutics* Vol. IV Pt. A, (Academic Press) pp. 225-274, and pp. 315.

Mott, N. F. 1952 *Phil. Mag.* **43**, 1151